Special Practice Problems

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~[JEE (Mains & Advanced)]~

Topic: Function

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. If $a\,f(x)+b\,f\left(\frac{1}{x}\right)=x-1,\ x\neq 0$ and $a\neq b$, then $f(2)$ is equal to
   (a) $\frac{a}{a^2-b^2}$ (b) $\frac{(a+2b)}{2(a^2-b^2)}$
   (c) $\frac{(a-2b)}{(a^2-b^2)}$ (d) $\frac{(2a+b)}{2(a^2-b^2)}$

2. If $f(x)=2x^n+a$, if $f(2)=26$ and $f(4)=138$, then $f(3)$ is equal to
   (a) 56 (b) 82
   (c) 64 (d) 122

3. The period of the function $f(x)=4\sin^4\left(\frac{4x-3\pi}{6\pi^2}\right)+2\cos\left(\frac{4x-3\pi}{3\pi^2}\right)$ is
   (a) $\frac{3\pi^2}{4}$ (b) $\frac{3\pi^3}{4}$
   (c) $\frac{4\pi^2}{3}$ (d) $\frac{4\pi^3}{3}$

4. Range of values of $f(x)=1+\sin x+\sin^3 x+\sin^5 x+\ldots;x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is
   (a) (0, 1) (b) (0,2)
   (c) (–2, 2) (d) $(-\infty,\infty)$

5. Range of the function f defined by $f(x)=\left[\frac{1}{\sin\{x\}}\right]$ (where [.] and {.} respectively denotes the greatest integer and the fractional part function) is
   (a) I, the set of integers
   (b) N, the set of natural numbers
   (c) W, the set of whole numbers
   (d) Q, the set of rational numbers

6. If $f(x)=\sqrt{3|x|-x-2}$ and $g(x)=\sin x$, then domain of definition of $(fog)\,x$ is
   (a) $\left\{2n\pi+\frac{\pi}{2}\right\}_{n\in I}$
   (b) $\bigcup_{n\in I}\left(2n\pi+\frac{7\pi}{6},2n\pi+\frac{11\pi}{6}\right)$
   (c) $\left\{2n\pi+\frac{7\pi}{6}\right\}_{n\in I}$
   (d) $\left\{(4m+1)\frac{\pi}{2},m\in I\right\}\bigcup_{n\in I}\left[2n\pi+\frac{7\pi}{6},2n\pi+\frac{11\pi}{6}\right]$

7. Let $f(x)=\frac{\sin 2nx}{1+\cos^2 nx}$, $n\in N$ has $\frac{\pi}{6}$ as its fundamental period, then n is equal to
   (a) 2 (b) 4
   (c) 6 (d) 8

8. Let $f(x)=[9^x-3^x+1]\,\forall\, x\in(-\infty,1)$, then range of $f(x)$ is ([.] denotes the greatest integer function)
   (a) {0, 1, 2, 3, 4, 5, 6} (b) {0, 1, 2, 3, 4, 5, 6, 7}
   (c) {1, 2, 3, 4, 5, 6} (d) {1, 2, 3, 4, 5, 6, 7}

9. Let $f:R\to R$, where $f(x)=\left(\frac{ax+5}{x^2+2}\right)$, then the values of '$a$' for which the function is invertiable is
   (a) $(0,\infty)$ (b) $(1,\infty)$
   (c) (0, 1) (d) none of these

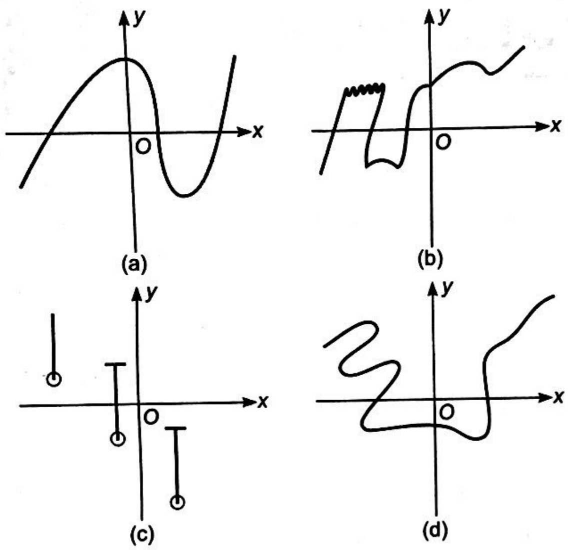
10. Period of the function $f(x)=\frac{\sin\{\sin(nx)\}}{\tan\left(\frac{x}{n}\right)}$, $n\in N$, is 6π, then n is equal to
    (a) 1 (b) 2
    (c) 3 (d) none of these

11. If $f(x)$ is an even function and satisfies the relation $x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x)$, where $g(x)$ is an odd function, then the value of $f(5)$ is

(a) 0 (b) $\frac{37}{75}$

(c) 4 (d) $\frac{51}{77}$

12. Which of the following graphs are graphs of functions



13. The graph of $f(x) = \left|\left|\left(\frac{1}{|x|} - n\right)\right| - n\right|$ is lie in the $(n > 0)$

(a) I and II quadrant (b) I and III quadrant

(c) I and IV quadrant (d) II and III quadrant

14. $f(x) = (\sin x^7)\, e^{x^{5\, Sgn x^9}}$ is

(a) an even function (b) an odd function

(c) neither even nor odd (d) none of these

15. Let $f(x) = \sqrt{([\sin 2x] - [\cos 2x])}$ (where [.] denotes the greatest integer function), then range of $f(x)$ will be

(a) $\{0\}$ (b) $\{1\}$

(c) $\{0, 1\}$ (d) $\{0, 1, \sqrt{2}\}$

16. If $f : [-20, 20] \to R$, defined by $f(x) = \left[\frac{x^2}{a}\right] \sin x + \cos x$, (where [.] denotes the greatest integer function) is an even function, then set of values of 'a' is given by

(a) null set (b) R

(c) $[0, 400]$ (d) $(400, \infty)$

17. Let $A \equiv \{1, 2, 3, 4\}, B \equiv \{a, b, c\}$, then number of functions from $A \to B$, which are not onto is

(a) 8 (b) 24

(c) 45 (d) 6

18. Let $f(x) = \sin^{-1} \sin (\tan x)$ and $g(x) = \cos^{-1} \sin \sqrt{(1 - \tan^2 x)}$ are same functions, then $x \in$

(a) $\left[0, \tan^{-1} \frac{\pi}{2}\right]$ (b) $[0, 1]$

(c) $[0, \infty]$ (d) none of these

19. If $f(x) + 2f(1 - x) = x^2 + 2, \forall x \in R$, then $f(x)$ is given as

(a) $\frac{(x-1)^2}{3}$ (b) $\frac{(x-2)^2}{3}$

(c) $x^2 - 1$ (d) $x^2 - 2$

20. If $2f(x - 1) - f\left(\frac{1-x}{x}\right) = x$, then $f(x)$ is

(a) $\frac{1}{3}\left\{2(1+x) + \frac{1}{(1+x)}\right\}$ (b) $2(x-1) - \frac{(1-x)}{x}$

(c) $x^2 + \frac{1}{x^2} + 4$ (d) $\frac{1}{4}\left\{(x+2) + \frac{1}{(x+2)}\right\}$

21. If $f : R \to R$ be a function satisfying $f(2x + 3) + f(2x + 7) = 2, \forall x \in R$, then period of $f(x)$ is

(a) 2 (b) 4

(c) 8 (d) 16

22. The range of the function $f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$, (where [.] denotes the greatest integer function) is

(a) $\left\{\frac{\pi}{2}\right\}$ (b) $\{\pi\}$

(c) $\left\{-\frac{1}{2}, 0\right\}$ (d) $\left(0, \frac{\pi}{2}\right)$

23. Total number of solutions of $2^x + 3^x + 4^x - 5^x = 0$ is

(a) 0 (b) 1

(c) 2 (d) infinitely many

24. If $2 < x^2 < 3$, then the number of positive roots of $\{x^2\} = \left\{\frac{1}{x}\right\}$, (where $\{x\}$ denotes the fractional part of x) is

(a) 0 (b) 1

(c) 2 (d) 3

25. If $f(x)$ and $g(x)$ are periodic functions with periods 7 and 11 respectively. Then the period of $F(x) = f(x)\, g\left(\frac{x}{5}\right) - g(x) f\left(\frac{x}{3}\right)$ is

(a) 177 (b) 222

(c) 433 (d) 1155

26. Let $A = \{1, 2, 3, 4, 5, 6\}$. If f be a bijective function from A to A, then the number of such functions for which $f(\lambda) \neq \lambda, \lambda = 1, 2, 3, 4, 5, 6$ is

(a) 44 (b) 265

(c) 325 (d) 4585

27. If $f(2x + 3y, 2x - 7y) = 20x$, then $f(x, y)$ equals

(a) $7x - 3y$ (b) $7x + 3y$

(c) $3x - 7y$ (d) $3x + 7y$

28. If $f(x) = -\dfrac{x|x|}{1+x^2}$, then $f^{-1}(x)$ equals

(a) $\sqrt{\dfrac{|x|}{1-|x|}}$ (b) $(Sgn\ x)\sqrt{\dfrac{|x|}{1-|x|}}$

(c) $-\sqrt{\dfrac{x}{1-x}}$ (d) none of these

29. Let $f: R \to R$ defined by $f(x) = \dfrac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$, then

(a) $f(x)$ is one-one but not onto
(b) $f(x)$ is neither one-one nor onto
(c) $f(x)$ is many one but onto
(d) $f(x)$ is one-one and onto

30. The function $f(x) = \lambda|\sin x| + \lambda^2|\cos x| + g(\lambda)$ has period equal to $\dfrac{\pi}{2}$, then λ is

(a) 2 (b) 1
(c) 3 (d) none of these

31. If f is decreasing odd function, then f^{-1} is

(a) odd and decreasing (b) odd and increasing
(c) even and decreasing (d) even and increasing

32. The range of the function $f(x) = 3|\sin x| - 2|\cos x|$ is

(a) $[-2, \sqrt{13}]$ (b) $[-2, 3]$
(c) $[3, \sqrt{13}]$ (d) $[-3, 2]$

33. The domain of the function $f(x) = \sqrt{\left(\dfrac{1}{\sin x} - 1\right)}$ is

(a) $\left(2n\pi, 2n\pi + \dfrac{\pi}{2}\right), \forall n \in I$
(b) $(2n\pi, (2n+1)\pi), \forall n \in I$
(c) $((2n-1)\pi, 2n\pi), \forall n \in I$
(d) none of the above

34. If $g(x) = [x^2] - [x]^2$, where [.] denotes the greatest integer function and $x \in [0, 2]$, then the set of values of $g(x)$ is

(a) $\{-1, 0\}$ (b) $\{-1, 0, 1\}$
(c) $\{0\}$ (d) $\{0, 1, 2\}$

35. Which of the following functions is periodic with period π ?

(a) $f(x) = \sin 3x$ (b) $f(x) = |\cos x|$
(c) $f(x) = [x + \pi]$ (d) $f(x) = x \cos x$

where [x] means the greatest integer not greater than x.

36. The domain of definition of $f(x) = \sqrt{\dfrac{1-|x|}{2-|x|}}$ is

(a) $(-\infty, \infty) - [-2, 2]$
(b) $(-\infty, \infty) - [-1, 1]$
(c) $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$
(d) none of the above

37. Let $f: R \to R$ be a given function and $A \subset R$ and $B \subset R$, then

(a) $f(A \cup B) = f(A) \cup f(B)$
(b) $f(A \cap B) = f(A) \cap f(B)$
(c) $f(A^c) = [f(A)]^c$
(d) $f(A/B) = f(A)/f(B)$

38. The domain of the function $y = \underbrace{\log_{10}\log_{10}\log_{10}\ldots\log_{10} x}_{n\text{ times}}$ is

(a) $[10^n, \infty)$ (b) $(10^{n-1}, \infty)$
(c) $(10^{n-2}, \infty)$ (d) none of these

39. If [x] and {x} represent integral and fractional parts of x, then the value of $\sum_{r=1}^{2000} \dfrac{\{x+r\}}{2000}$ is

(a) x (b) $[x]$
(c) $\{x\}$ (d) $x + 2001$

40. If [.] denotes the greatest integer function, then the value of $\sum_{r=1}^{100}\left[\dfrac{1}{2} + \dfrac{r}{100}\right]$ is

(a) 49 (b) 50
(c) 51 (d) 52

41. If $f(x)$ is a polynomial satisfying $f(x)\cdot f(1/x) = f(x) + f(1/x)$ and $f(3) = 28$, then $f(4)$ is equal to

(a) 63 (b) 65
(c) 17 (d) none of these

42. If $f(x+y) = f(x) + f(y) - xy - 1$ for all x, y and $f(1) = 1$, then the number of solutions of $f(n) = n, n \in N$ is

(a) one (b) two
(c) three (d) none of these

43. The function $f(x) = \sin\left(\dfrac{\pi x}{n!}\right) - \cos\left(\dfrac{\pi x}{(n+1)!}\right)$ is

(a) non periodic
(b) periodic, with period $2(n!)$
(c) periodic, with period $(n+1)$
(d) none of the above

44. The value of b and c for which the identity $f(x+1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$ are

(a) $b = 2, c = 1$ (b) $b = 4, c = -1$
(c) $b = -1, c = 4$ (d) $b = -1, c = 1$

45. The value of the parameter α, for which the function $f(x) = 1 + \alpha x, \alpha \neq 0$ is the inverse of itself, is

(a) -2 (b) -1
(c) 1 (d) 2

46. Which of the following function is even function

(a) $f(x) = \left(\dfrac{a^x + 1}{a^x - 1}\right)$ (b) $f(x) = x\left(\dfrac{a^x - 1}{a^x + 1}\right)$

(c) $f(x) = \left(\dfrac{a^x - a^{-x}}{a^x + a^{-x}}\right)$ (d) $f(x) = \sin x$

47. If S is the set of all real x for which $1 - e^{(1/x)-1} > 0$, then S is equal to

(a) $(-\infty, 0) \cup (1, \infty)$ (b) $(-\infty, \infty)$
(c) $(-\infty, 0] \cup [1, \infty)$ (d) none of these

48. If $f(x) = \sin^2 x + \sin^2\left(x + \dfrac{\pi}{3}\right) + \cos x \cdot \cos\left(x + \dfrac{\pi}{3}\right)$ and $g(5/4) = 1$, then $(gof)\,x$ is

(a) a polynomial of the first degree in $\sin x$, $\cos x$
(b) a constant function
(c) a polynomial of the second degree in $\sin x$, $\cos x$
(d) none of the above

49. If the function $f:[1,\infty)\to[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$, then $f^{-1}(x)$ is

(a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}(1+\sqrt{(1+4\log_2 x)})$

(c) $\frac{1}{2}(1-\sqrt{(1+4\log_2 x)})$ (d) not defined

50. Let f be a function satisfying $2f(xy)=\{f(x)\}^y+\{f(y)\}^x$ and $f(1)=k\neq 1$, then $\sum_{r=1}^{n} f(r)$ is equal to

(a) k^n-1 (b) k^n

(c) k^n+1 (d) none of these

51. Which one of the following functions are periodic?

(a) $f(x)=x-[x]$, where $[x]\le x$

(b) $f(x)=x\sin(1/x)$ for $x\neq 0$, $f(0)=0$

(c) $f(x)=x\cos x$

(d) None of the above

52. The domain of the function $f(x)=1/\log_{10}(1-x)+\sqrt{(x+2)}$ is

(a) $[-3,-2]$, excluding (-2.5)

(b) $[0, 1]$, excluding 0.5

(c) $[-2, 1]$, excluding 0

(d) none of the above

53. The graph of the function $y=f(x)$ is symmetrical about the line $x=2$, then

(a) $f(x+2)=f(x-2)$ (b) $f(2+x)=f(2-x)$

(c) $f(x)=f(-x)$ (d) none of these

54. The range of the function $f(x)=6^x+3^x+6^{-x}+3^{-x}+2$ is

(a) $[-2,\infty)$ (b) $(-2,\infty)$

(c) $(6,\infty)$ (d) $[6,\infty)$

55. If $f:X\to Y$ defined by $f(x)=\sqrt{3}\sin x+\cos x+4$ is one-one and onto, then Y is

(a) $[1, 4]$ (b) $[2, 5]$

(c) $[1, 5]$ (d) $[2, 6]$

56. If $f(x)=\cos^{-1}(x-x^2)+\sqrt{\left(1-\frac{1}{|x|}\right)}+\frac{1}{[x^2-1]}$, then domain of $f(x)$ is (where [.] is the greatest integer)

(a) $\left(\sqrt{2},\frac{1+\sqrt{5}}{2}\right]$ (b) $\left(-\sqrt{2},\frac{1-\sqrt{5}}{2}\right)$

(c) $\left[\sqrt{2},\frac{1+\sqrt{5}}{2}\right]$ (d) none of these

57. The range of function $f:[0,1]\to R$, $f(x)=x^3-x^2+4x+2\sin^{-1}x$ is

(a) $[-\pi-2, 0]$ (b) $[2, 3]$

(c) $[0, 4+\pi]$ (d) $(0, 2+\pi]$

58. Let $f(x)$ be a function defined on $[0, 1]$ such that

$$f(x)=\begin{cases} x, & \text{if } x\in Q \\ 1-x, & \text{if } x\notin Q. \end{cases}$$

Then for all $x\in[0,1]$ $(f0f)(x)$ is

(a) constant (b) $1+x$

(c) x (d) none of these

59. If $f:R\to R$ is a function such that $f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3)$ for all $x\in R$, the $f(2)-f(1)$

(a) $f(0)$ (b) $-f(0)$

(c) $f'(0)$ (d) $-f'(0)$

60. Let $f:R\to Q$ be a continuous function such that $f(2)=3$, then

(a) $f(x)$ is always an even function

(b) $f(x)$ is always an odd function

(c) nothing can be said about $f(x)$ being even or odd

(d) $f(x)$ is an increasing function

61. The greatest value of the function $f(x)=\cos\{xe^{[x]}+2x^2-x\}$, $x\in(-1,\infty)$, where $[x]$ denotes the greatest integer less than or equal to x is

(a) 0 (b) 1

(c) 2 (d) 3

62. The period of $e^{\cos^4\pi x+x-[x]+\cos^2\pi x}$ is ([.] denotes the greatest integer function)

(a) 2 (b) 1

(c) 0 (d) -1

63. If $f(x)=\sin^{-1}\{4-(x-7)^3\}^{1/5}$, then its inverse is

(a) $(4-\sin^5 x)^{1/3}$ (b) $7-(4-\sin^5 x)^{1/3}$

(c) $(4-\sin^5 x)^{2/3}$ (d) $7+(4-\sin^5 x)^{1/3}$

64. The period of $f(x)=\frac{1}{2}\left\{\frac{|\sin x|}{\cos x}+\frac{|\cos x|}{\sin x}\right\}$ is

(a) 2π (b) π

(c) $\pi/2$ (d) $\pi/4$

65. Given $f(x)=\frac{1}{(1-x)}$, $g(x)=f\{f(x)\}$ and $h(x)=f[f\{f(x)\}]$. Then the value of $f(x)\cdot g(x)\cdot h(x)$ is

(a) 0 (b) -1

(c) 1 (d) 2

66. The inverse of the function $y=\log_a(x+\sqrt{(x^2+1)})$; $(a>0, a\neq 1)$ is

(a) $\frac{1}{2}(a^x-a^{-x})$

(b) not defined for all x

(c) defined for only positive x

(d) none of the above

67. The domain of the function $f(x)=\sqrt{\sin^{-1}(\log_2 x)}+\sqrt{\cos(\sin x)}+\sin^{-1}\left(\frac{1+x^2}{2x}\right)$

(a) $\{x:1\le x\le 2\}$

(b) $\{1\}$

(c) not defined for any value of x

(d) $\{-1, 1\}$

68. Let $f : R \to R$ be a function defined by $f(x) = \dfrac{x^2 + 2x + 5}{x^2 + x + 1}$ is
(a) one-one and into (b) one-one and onto
(c) many one and onto (d) many one and into

69. Let $f(x) = \begin{cases} 1 + x, & 0 \le x \le 2 \\ 3 - x, & 2 < x \le 3 \end{cases}$, then $f0f(x)$
(a) $= \begin{cases} 2 + x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \\ 4 - x, & 2 < x \le 3 \end{cases}$ (b) $= \begin{cases} 2 + x, & 0 \le x \le 2 \\ 4 - x, & 2 < x \le 3 \end{cases}$
(c) $= \begin{cases} 2 + x, & 0 \le x \le 2 \\ 2 - x, & 2 < x \le 3 \end{cases}$ (d) none of these

70. Let $f : R \to R, g : R \to R$ be two given functions such that f is injective and g is surjective, then which of the following is injective
(a) gof (b) fog
(c) gog (d) fof

71. The domain of $f(x) = \dfrac{1}{\sqrt{|\cos x| + \cos x}}$ is
(a) $[-2n\pi, 2n\pi]; \forall n \in I$ (b) $(2n\pi, \overline{2n + 1}\,\pi); \forall n \in I$
(c) $\left(\dfrac{(4n+1)\pi}{2}, \dfrac{(4n+3)\pi}{2}\right); \forall n \in I$
(d) $\left(\dfrac{(4n-1)\pi}{2}, \dfrac{(4n+1)\pi}{2}\right); \forall n \in I$

72. The domain of the function $f(x) = {}^{16-x}C_{2x-1} + {}^{20-3x}P_{4x-5}$, where the symbols have their usual meanings, is the set
(a) $\{2, 3\}$ (b) $\{2, 3, 4\}$
(c) $\{1, 2, 3, 4, 5\}$ (d) none of these

73. If $f(x) = 3\sin\sqrt{\left(\dfrac{\pi^2}{16} - x^2\right)}$, then its range is
(a) $\left[-\dfrac{3}{\sqrt{2}}, \dfrac{3}{\sqrt{2}}\right]$ (b) $\left[0, \dfrac{3}{\sqrt{2}}\right]$
(c) $\left[-\dfrac{3}{\sqrt{2}}, 0\right]$ (d) none of these

74. The domain of
$f(x) = \sqrt{\{x - 4 - 2\sqrt{(x-5)}\}} - \sqrt{\{x - 4 + 2\sqrt{x-5}\}}$ is
(a) $[-5, \infty)$ (b) $(-\infty, 2]$
(c) $[5, \infty)$ (d) none of these

75. The period of $\dfrac{|\sin x| + |\cos x|}{|\sin x - \cos x|}$ is
(a) 2π (b) π
(c) $\pi/2$ (d) $\pi/4$

76. If [.] denotes the greatest integer function, then the domain of the real valued function $\log_{[x+1/2]}|x^2 - x - 2|$ is
(a) $\left[\dfrac{3}{2}, \infty\right)$ (b) $\left[\dfrac{3}{2}, 2\right) \cup (2, \infty)$
(c) $\left(\dfrac{1}{2}, 2\right) \cup (2, \infty)$ (d) none of these

77. Let $f(x) = \sin^2(x/2) + \cos^2(x/2)$ and $g(x) = \sec^2 x - \tan^2 x$. The two functions are equal over the set
(a) ϕ
(b) R
(c) $R - \left\{x : x = (2n+1)\dfrac{\pi}{2}, n \in I\right\}$
(d) none of the above

78. The domain of $f(x)$ is (0, 1), therefore domain of $f(e^x) + f(\ln|x|)$ is
(a) $(-1, e)$ (b) $(1, e)$
(c) $(-e, -1)$ (d) $(-e, 1)$

79. If $f : [-4, 0] \to R$ is defined by $e^x + \sin x$, its even extension to $[-4, 4]$ is given by
(a) $-e^{-|x|} - \sin|x|$ (b) $e^{-|x|} - \sin|x|$
(c) $e^{-|x|} + \sin|x|$ (d) $-e^{-|x|} + \sin|x|$

80. If $g(x)$ be a function defined on $[-1, 1]$ if the area of the equilateral triangle with two of its vertices at (0, 0) and $(x, g(x))$ is $\sqrt{3}/4$, then
(a) $g(x) = \pm\sqrt{(1 - x^2)}$ (b) $g(x) = -\sqrt{(1 - x^2)}$
(c) $g(x) = \sqrt{(1 - x^2)}$ (d) $g(x) = \sqrt{(1 + x^2)}$

81. The period of the function
$$f(x) = a^{\sin^2 x + \sin^2(x + \pi/3) + \cos x \cos(x + \pi/3)}$$ is
(where a is constant)
(a) 1 (b) $\pi/2$
(c) π (d) cannot be determined

82. The domain of the function
$$f(x) = \sin^{-1}\left(\frac{2 - |x|}{4}\right) + \cos^{-1}\left(\frac{2 - |x|}{4}\right) + \tan^{-1}\left(\frac{2 - |x|}{4}\right)$$ is
(a) $[0, 3]$ (b) $[-6, 6]$
(c) $[-1, 1]$ (d) $[-3, 3]$

83. Let f be a real valued function defined by $f(x) = \dfrac{e^x - e^{-|x|}}{e^x + e^{|x|}}$, then range of f is
(a) R (b) $[0, 1]$
(c) $[0, 1)$ (d) $[0, 1/2)$

84. If $f(x)$ is a polynomial function of the second degree such that $f(-3) = 6$, $f(0) = 6$ and $f(2) = 11$, then the graph of the function $f(x)$ cuts the ordinate $x = 1$ at the point
(a) (1, 8) (b) (1, – 2)
(c) 1, 4 (d) none of these

85. If $f(x + y, x - y) = xy$, then the arithmetic mean of $f(x, y)$ and $f(y, x)$ is
(a) x (b) y
(c) 0 (d) none of these

86. Under the condition……,the domain of $f_1 + f_2$ is equal to dom $f_1 \cup$ dom f_2
(a) dom $f_1 \ne$ dom f_2 (b) dom $f_1 =$ dom f_2
(c) dom $f_1 >$ dom f_2 (d) dom $f_1 <$ dom f_2

87. If the function $f: R \to R$ be such that $f(x) = x - [x]$, where [.] denotes the greatest integer function, then $f^{-1}(x)$ is

(a) $\dfrac{1}{x-[x]}$ (b) $[x]-x$

(c) not defined (d) none of these

88. The domain of the function

$$f(x) = \sqrt{(2-|x|)} + \sqrt{(1+|x|)}$$

(a) [2, 6] (b) (– 2, 6]

(c) [8, 12] (d) none of these

89. Let $f: R \to [0, \pi/2)$ be a function defined by $f(x) = \tan^{-1}(x^2 + x + a)$. If f is onto, then a equals

(a) 0 (b) 1

(c) 1/2 (d) 1/4

90. Let $f(x) = \cos\sqrt{k}\,x$, where $k = [m]$ = the greatest integer $\le m$, if the period of $f(x)$ is π, then

(a) $m \in [4, 5)$ (b) $m = 4, 5$

(c) $m \in [4, 5]$ (d) none of these

91. Domain of $\sin^{-1}[\sec x]$ ([.] is greatest integer less than or equal to x) is

(a) $\{(2n+1)\pi, (2n+9)\pi\} \cup \{[(2m-1)\pi, 2m\pi + \pi/3), m \in I\}$

(b) $\{2n\pi, n \in I\} \cup \{[2m\pi, (2m+1)\pi, m \in I\}$

(c) $\{(2n+1)\pi, n \in I\} \cup \{[2m\pi, 2m\pi + \pi/3], m \in I\}$

(d) none of the above

92. Let $f(x) = (x^{12} - x^9 + x^4 - x + 1)^{-1/2}$. The domain of the function is

(a) $(-\infty, -1)$ (b) $(-1, 1)$

(c) $(1, \infty)$ (d) $(-\infty, \infty)$

93. The function $f(x) = \int_0^x \log_e\left(\dfrac{1-x}{1+x}\right)dx$ is

(a) an even function

(b) an odd function

(c) a periodic function

(d) none of these

94. If $f: R \to R, g: R \to R$ be two given functions, then $f(x) = 2\min(f(x)-g(x), 0)$ equals

(a) $f(x)+g(x)-|g(x)-f(x)|$

(b) $f(x)+g(x)+|g(x)-f(x)|$

(c) $f(x)-g(x)+|g(x)-f(x)|$

(d) $f(x)-g(x)-|g(x)-f(x)|$

95. The domain of the function $f(x) = \ln\left(\ln\dfrac{x}{\{x\}}\right)$ is (where {.} denotes the fractional part function)

(a) $(0, \infty) - I$ (b) $(1, \infty) - I$

(c) $R - I$ (d) $(2, \infty) - I$

96. $\sin ax + \cos ax$ and $|\sin x| + |\cos x|$ are periodic of same fundamental period, if a equals

(a) 0 (b) 1

(c) 2 (d) 4

97. If $g(x)$ is a polynomial satisfying $g(x)g(y) = g(x) + g(y) + g(xy) - 2$ for all real x and y and $g(2) = 5$, then $g(3)$ is equal to

(a) 10 (b) 24

(c) 21 (d) none of these

98. The interval into which the function $y = \dfrac{(x-1)}{(x^2-3x+3)}$ transforms the entire real line is

(a) $\left[\dfrac{1}{3}, 2\right]$ (b) $\left[-\dfrac{1}{3}, 1\right]$

(c) $\left[-\dfrac{1}{3}, 2\right]$ (d) none of these

99. Let the function $f(x) = x^2 + x + \sin x - \cos x + \log(1+|x|)$ be defined over the interval [0, 1]. The odd extension of $f(x)$ in the interval [– 1, 1] is

(a) $x^2 + x + \sin x + \cos x - \log(1+|x|)$

(b) $-x^2 + x + \sin x + \cos x - \log(1+|x|)$

(c) $-x^2 + x + \sin x - \cos x + \log(1+|x|)$

(d) none of the above

100. The function $f(x) = \dfrac{\sec^{-1}x}{\sqrt{x-[x]}}$, where [x] denotes the greatest integer less than or equal to x is defined for all x belonging to

(a) R

(b) $R - \{(-1, 1) \cup \{n, n \in I\}\}$

(c) $R^+ - (0, 1)$

(d) $R^+ - \{n, n \in N\}$

101. The period of the function $f(x) = [\sin 3x] + |\cos 6x|$ is ([.] denotes the greatest integer less than or equal to x)

(a) π (b) $2\pi/3$

(c) 2π (d) none of these

Objective Questions Type II [One or more than one correct answer(s)]

In each of the questions below four choices of which one or more than one are correct. You have to select the correct answer(s) accordingly.

1. If $f(x)=\dfrac{x}{x^2+1}$ and $f(A)=\left\{y:-\dfrac{1}{2}\le y<0\right\}$, then set A is
(a) $[-1, 0)$ (b) $(-\infty, -1]$
(c) $(-\infty, 0)$ (d) $(-\infty, \infty)$

2. Let $f(x)=2x-\sin x$ and $g(x)=\sqrt[3]{x}$, then
(a) range of gof is R
(b) gof is one-one
(c) both f and g are one-one
(d) both f and g are onto

3. Let $f(x)=\begin{cases}0, & \text{for } x=0\\ x^2\sin\left(\dfrac{\pi}{x}\right), & \text{for } -1<x<1,\ (x\neq 0),\\ x|x|, & \text{for } x\ge 1 \text{ or } x\le -1\end{cases}$
then
(a) $f(x)$ is an odd function
(b) $f(x)$ is an even function
(c) $f(x)$ is neither odd nor even
(d) $f'(x)$ is an even function

4. Which of the following function is periodic
(a) $Sgn\,(e^{-x})$
(b) $\sin x+|\sin x|$
(c) $\min(\sin x, |x|)$
(d) $\left[x+\dfrac{1}{2}\right]+\left[x-\dfrac{1}{2}\right]+2[-x]$
([x] denotes the greatest integer function)

5. Of the following functions defined from $[-1, 1]$ to $[-1, 1]$ select those which are not bijective
(a) $\sin(\sin^{-1}x)$ (b) $\dfrac{2}{\pi}\sin^{-1}(\sin x)$
(c) $(Sgn\,x)/\ln(e^x)$ (d) $x^3(Sgn\,x)$

6. If $[x]$ denotes the greatest integer less than or equal to x, the extreme values of the function $f(x)=[1+\sin x]+[1+\sin 2x]+[1+\sin 3x]+\ldots+[1+\sin nx]$, $n\in I^+$, $x\in(0,\pi)$ are
(a) $n-1$ (b) n
(c) $n+1$ (d) $n+2$

7. Domain of $f(x)=\sin^{-1}[2-4x^2]$ is ([.] denotes the greatest integer function)
(a) $\left[-\dfrac{\sqrt{3}}{2}, \dfrac{\sqrt{3}}{2}\right]\sim\{0\}$ (b) $\left[-\dfrac{\sqrt{3}}{2}, 0\right)$
(c) $\left[-\dfrac{\sqrt{3}}{2}, 0\right)\cup\left(0, \dfrac{\sqrt{3}}{2}\right]$ (d) $\left[-\dfrac{\sqrt{3}}{2}, 8\right]$

8. If $e^x+e^{f(x)}=e$, then for $f(x)$
(a) domain $=(-\infty, 1)$ (b) range $=(-\infty, 1)$
(c) domain $=(-\infty, 0]$ (d) range $=(-\infty, 1]$

9. Let $f(x)=\sec^{-1}[1+\cos^2 x]$, where [.] denotes the greatest integer function, then
(a) the domain of f is R
(b) the domain of f is $[1, 2]$
(c) the range of f is $[1, 2]$
(d) the range of f is $\{\sec^{-1}1, \sec^{-1}2\}$

10. Let $f(x)=[x]^2+[x+1]-3$, where $[x]\le x$. Then
(a) $f(x)$ is a many-one and into function
(b) $f(x)=0$ for infinite number of values of x
(c) $f(x)=0$ for only two real values
(d) none of the above

11. If domain of f is D_1 and domain of g is D_2, then domain of $f+g$ is
(a) D_1/D_2 (b) $D_1-(D_1/D_2)$
(c) $D_2/(D_2/D_1)$ (d) $D_1\cap D_2$

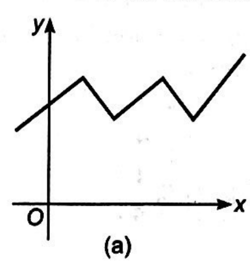
12. If $y=f(x)=\dfrac{x+2}{x-1}$, then
(a) $x=f(y)$
(b) $f(1)=3$
(c) y increases with x for $x<1$
(d) f is rational function of x

13. If $f(x)=\cos([\pi^2]x)+\cos([-\pi^2]x)$, where $[x]$ stands for the greatest integer function, then
(a) $f\left(\dfrac{\pi}{2}\right)=-1$ (b) $f(\pi)=1$
(c) $f(-\pi)=0$ (d) $f\left(\dfrac{\pi}{4}\right)=1$

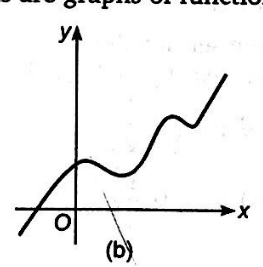
14. $f(x)=\cos^2 x+\cos^2\left(\dfrac{\pi}{3}+x\right)-\cos x\cos\left(\dfrac{\pi}{3}+x\right)$ is
(a) an odd function (b) an even function
(c) a periodic function (d) $f(0)=f(1)$

15. The possible values of 'a' for which the function $f(x)=e^{x-[x]}+\cos ax$ (where [.] denotes the greatest integer function) is periodic with finite fundamental period is
(a) π (b) 2π
(c) 3π (d) 1

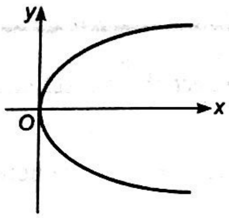
16. Which of the following graphs are graphs of functions



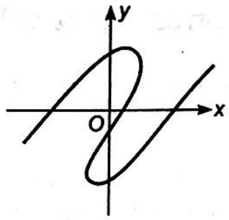
(a)



(b)



(c)



(d)

17. If $f(x)=\left(\dfrac{x-1}{x+1}\right)$, then which of the following statement(s) is/are correct

(a) $f\left(\dfrac{1}{x}\right)=f(x)$ (b) $f\left(\dfrac{1}{x}\right)=-f(x)$

(c) $f\left(-\dfrac{1}{x}\right)=\dfrac{1}{f(x)}$ (d) $f\left(-\dfrac{1}{x}\right)=-\dfrac{1}{f(x)}$

18. Let f be the greatest integer function and g be the modulus functions, then

(a) $(gof-fog)\left(-\dfrac{5}{3}\right)=1$ (b) $(f+2g)(-1)=1$

(c) $(gof-fog)\left(\dfrac{5}{3}\right)=0$ (d) $(f+2g)(1)=1$

19. Which of the following functions are periodic?

(a) $f(x)=\sin x+|\sin x|$

(b) $g(x)=\dfrac{(1+\sin x)(1+\sec x)}{(1+\cos x)(1+\operatorname{cosec} x)}$

(c) $h(x)=\max(\sin x,\cos x)$

(d) $p(x)=[x]+\left[x+\dfrac{1}{3}\right]+\left[x+\dfrac{2}{3}\right]-3x+10$, where [.] denotes the greatest integer function.

20. Which of the following functions are even?

(a) $f(x)=x\left(\dfrac{a^x+1}{a^x-1}\right)$

(b) $g(x)=\ln(x+\sqrt{(x^2+a^2)})$

(c) $h(x)=\sqrt[3]{(1-x)^2}+\sqrt[3]{(1+x)^2}$

(d) $p(x)=\begin{cases}0, & \text{if } x \text{ is rational}\\ 1, & \text{if } x \text{ is irrational}\end{cases}$

21. Which of the following functions are not identical ?

(a) $f(x)=\dfrac{x}{x^2}$ and $g(x)=\dfrac{1}{x}$

(b) $f(x)=\dfrac{x^2}{x}$ and $g(x)=x$

(c) $f(x)=\ln x^4$ and $g(x)=4\ln x$

(d) $f(x)=\ln\{(x-1)(x-2)\}$ and $g(x)=\ln(x-2)+\ln(x-3)$

22. Let $f(x)=\dfrac{5\sqrt{\sin x}}{1+\sqrt[3]{\sin x}}$. If D is the domain of f, then D contains

(a) $(0,\pi)$ (b) $(-2\pi,-\pi)$

(c) $(2\pi,3\pi)$ (d) $(4\pi,6\pi)$

Linked-Comprehension Type

In these questions, a passage (paragraph) has been given followed by questions based on each of the passages. You have to answer the questions based on the passage given.

PASSAGE 1

Let $f(x)=x^2-5x+6$, $g(x)=f(|x|)$, $h(x)=|g(x)|$ and $\phi(x)=h(x)-(x)$ are four functions, where (x) is the least integral function of $x\ge x$.

On the basis of above information, answer the following questions :

1. The number of solutions of the equation $g(x)=0$ is

(a) 0 (b) 2

(c) 4 (d) 6

2. The value of λ for which the equation $g(x)-\lambda=0$ has exactly three real and distinct roots

(a) 2 (b) 4

(c) 6 (d) none of these

3. The set of values of μ such that the equation $h(x)-\mu=0$ has exactly eight real and distinct roots

(a) $\mu\in\left(0,\dfrac{1}{2}\right)$ (b) $\mu\in\left(0,\dfrac{1}{4}\right)$

(c) $\mu\in\left(0,\dfrac{1}{2}\right]$ (d) $\mu\in\left(0,\dfrac{1}{4}\right]$

4. The set of all values of x, such that equation $g(x)+|g(x)|=0$ is satisfied

(a) $[-3,-2]$ (b) $[2,3]$

(c) $[-3,-2]\cup[2,3]$ (d) ϕ

5. Which statement is correct for $\phi(x)=0$

(a) one value of x is satisfied for $\phi(x)=0$ and that x lie between 4 and 5

(b) one value of x is satisfied for $\phi(x)=0$ and that x lie between 3 and 4

(c) two values of x is satisfied for $\phi(x)=0$

(d) none of the above

PASSAGE 2

Let $f(x) = \min\{x-[x], -x-[-x]\}, -2 \le x \le 2$; $g(x) = |2-|x-2||, -2 \le x \le 2$ and $h(x) = \dfrac{|\sin x|}{\sin x}, -2 \le x \le 2$ and $x \ne 0$

(where $[x]$ denotes the greatest integer function $\le x$).

On the basis of above information, answer the following questions :

1. The number of solutions of the equation $x^2 + [f(x)]^2 = 1$ is $\{-1 \le x \le 1\}$
   (a) 0 (b) 2
   (c) 4 (d) 6
2. The range of $f(x)$ is
   (a) $\left[0, \frac{1}{2}\right]$ (b) $[0, 1]$
   (c) $[0, 2]$ (d) none of these
3. The sum of all the roots of the equation $g(x) - h(x) = 0$ is $\{-2 \le x \le 2\}$
   (a) positive (b) negative
   (c) zero (d) none of these
4. The set of values of a such that the equation $f(x) - a = 0$ has exactly eight real and distincts roots
   (a) $a \in \left(0, \frac{1}{2}\right)$ (b) $a \in \left[0, \frac{1}{2}\right)$
   (c) $a \in [0,1)$ (d) $a \in (0, 1)$
5. The value of $\int_{-2}^{2} f(x)\,dx$ is
   (a) 0 (b) 1
   (c) 2 (d) 8

PASSAGE 3

Let f be a function satisfying $f(x) = \dfrac{a^x}{a^x + \sqrt{a}} = g_a(x) \ (a > 0)$

On the basis of above information, answer the following questions :

1. Let $f(x) = g_9(x)$, then the value of $\left[\sum_{r=1}^{1995} f\left(\frac{r}{1996}\right)\right]$ is (where [.] denotes the greatest integer function)
   (a) 995 (b) 996
   (c) 997 (d) 998
2. Let $f(x) = g_4(x)$, then $\sum_{r=1}^{1996} f\left(\frac{r}{1997}\right)$ is
   (a) zero (b) even
   (c) odd (d) none of these
3. The value of $g_5(x) + g_5(1-x)$ is
   (a) 1 (b) 5
   (c) 10 (d) none of these
4. The value of $\sum_{r=1}^{2n-1} 2f\left(\frac{r}{2n}\right)$ is
   (a) 0 (b) $2n-1$
   (c) $2n$ (d) none of these
5. If the value of $\sum_{r=0}^{2n} f\left(\frac{r}{2n+1}\right) = \frac{1}{1+\sqrt{a}} + 987$, then the value of n is
   (a) 493 (b) 494
   (c) 987 (d) 988

PASSAGE 4

Let $F(x) = f(x) + g(x), G(x) = f(x) - g(x)$ and $H(x) = \dfrac{f(x)}{g(x)}$, where $f(x) = 1 - 2\sin^2 x$ and $g(x) = \cos 2x$, $\forall\ f : R \to [-1, 1]$ and $g : R \to [-1, 1]$.

On the basis of above information, answer the following questions :

1. Domain and range of $H(x)$ are respectively
   (a) R and $\{1\}$
   (b) R and $\{0, 1\}$
   (c) $R \sim \{(2n+1)\frac{\pi}{4}\}$, and $\{1\}, n \in I$
   (d) $R \sim \left\{(2n+1)\frac{\pi}{2}\right\}$, and $\{0, 1\}, n \in I$
2. If $F : R \to [-2, 2]$, then
   (a) $F(x)$ is one-one function
   (b) $F(x)$ is onto function
   (c) $F(x)$ is into function
   (d) none of the above

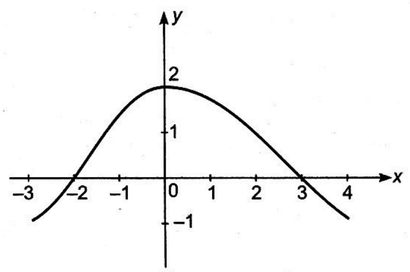
3. Which statement is correct?
(a) period of $f(x)$, $g(x)$ and $F(x)$ makes are AP with common difference $\pi/3$
(b) period of $f(x)$, $g(x)$ and $F(x)$ are same and is equal to 2π
(c) sum of periods of $f(x)$, $g(x)$ and $F(x)$ is 3π
(d) sum of periods of $f(x)$, $g(x)$ and $F(x)$ is 6π

4. Which statement is correct
(a) the domain of $G(x)$ and $H(x)$ are same
(b) the range of $G(x)$ and $H(x)$ are same
(c) the union of domain of $G(x)$ and $H(x)$ are all real numbers
(d) the union of domain of $G(x)$ and $H(x)$ are rational numbers

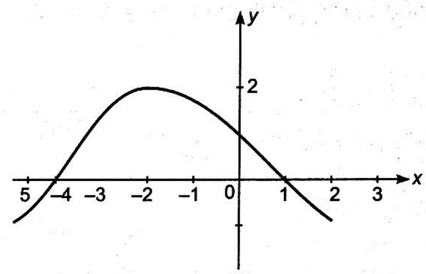
5. If the solutions of $F(x) - G(x) = 0$ are $x_1, x_2, x_3, \ldots, x_n$ where $x \in [0, 5\pi]$, then
(a) $x_1, x_2, x_3, \ldots, x_n$ are in AP with common difference $\pi / 4$
(b) the number of solutions of $F(x) - G(x) = 0$ is 10, $\forall\, x \in [0, 5\pi]$
(c) the sum of all solutions of $F(x) - G(x) = 0$, $\forall$ $x \in [0, 5\pi]$ is 25π
(d) (b) and (c) are correct

PASSAGE 5

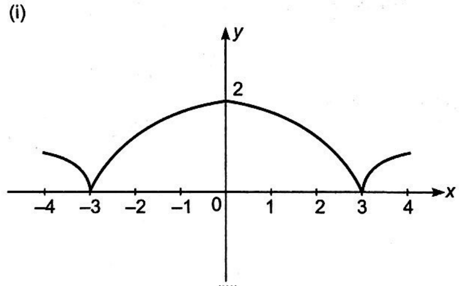
The accompanying figure shows the graph of a function $f(x)$ with domain $[-3, 4]$ and range $[-1, 2]$.



(i)



(ii)



(iii)

On the basis of above information, answer the following questions :

1. Figure (ii) represents the graph of the function
(a) $f(x)$ (b) $f(x-2)$
(c) $f(x+2)$ (d) $f(x-1)+1$

2. Figure (iii) represents the graph of the function
(a) $f(x)$ (b) $f(|x|)$
(c) $|f(x)|$ (d) $|f(|x|)|$

3. The domain and range respectively of
(a) $f(-x)$ are $[-4, 3]$ and $[-2, 1]$
(b) $f(x)-1$ are $[-3, 4]$ and $[-1, 2]$
(c) $f(x)+2$ are $[-3, 4]$ and $[-2, 4]$
(d) $-f(x+1)+1$ are $[-4, 3]$ and $[-1, 2]$

4. $[-2, 5]$ and $[-2, 1]$ are the domain and range respectively of the function
(a) $-f(x)$ (b) $f(x-1)$
(c) $-f(x+1)+1$ (d) $-f(x+1)$

5. The number of solutions of figure (iii) and $(2x-6)^2 + 4y^2 = 49$ are
(a) 2 (b) 4
(c) 6 (d) none of these

Answers

Objective Questions Type I [Only one correct answer]

1. (d)	2. (c)	3. (b)	4. (d)	5. (b)	6. (d)	7. (c)	8. (c)	9. (d)	10. (c)
11. (a)	12. (a)	13. (a)	14. (b)	15. (c)	16. (d)	17. (c)	18. (a)	19. (b)	20. (a)
21. (c)	22. (b)	23. (b)	24. (b)	25. (d)	26. (b)	27. (b)	28. (d)	29. (b)	30. (b)
31. (a)	32. (d)	33. (b)	34. (d)	35. (b)	36. (c)	37. (d)	38. (d)	39. (c)	40. (c)
41. (b)	42. (a)	43. (d)	44. (b)	45. (b)	46. (b)	47. (a)	48. (b)	49. (b)	50. (d)
51. (a)	52. (d)	53. (b)	54. (d)	55. (d)	56. (c)	57. (c)	58. (c)	59. (b)	60. (a)
61. (b)	62. (b)	63. (d)	64. (a)	65. (b)	66. (a)	67. (b)	68. (d)	69. (a)	70. (d)
71. (d)	72. (a)	73. (b)	74. (c)	75. (b)	76. (b)	77. (c)	78. (c)	79. (b)	80. (a)
81. (d)	82. (b)	83. (d)	84. (a)	85. (c)	86. (b)	87. (c)	88. (d)	89. (d)	90. (a)
91. (c)	92. (d)	93. (a)	94. (d)	95. (b)	96. (d)	97. (a)	98. (b)	99. (b)	100. (b)
101. (b)									

Objective Questions Type II [One or more than one correct answer(s)]

1. (a, b, c)	2. (a, b, c, d)	3. (a, d)	4. (a, b, c, d)	5. (b, c, d)
6. (b, c)	7. (a, c)	8. (a, b)	9. (a, d)	10. (a, b)
11. (b, c, d)	12. (a, d)	13. (a, c)	14. (b, c, d)	15. (a, b, c)
16. (a, b)	17. (b, d)	18. (a, b, c)	19. (a, b, c, d)	20. (a, c, d)
21. (b, c, d)	22. (a, b, c)			

Linked-Comprehension Type

Passage 1	1. (c)	2. (c)	3. (b)	4. (c)	5. (b)
Passage 2	1. (b)	2. (a)	3. (a)	4. (a)	5. (b)
Passage 3	1. (c)	2. (b)	3. (a)	4. (b)	5. (c)
Passage 4	1. (c)	2. (b)	3. (c)	4. (c)	5. (d)
Passage 5	1. (c)	2. (d)	3. (d)	4. (d)	5. (d)